Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(OLD) EXAMINATION – SUMMER 2019

Subject Code: 130002 **Date: 30/05/2019**

Subject Name: Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Solve $3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$ **03**

(ii) Solve $y' - (1 + 3x^{-1})y = x + 2;\ y(1) = e - 1$ **04**

(b) Find the Power series solution of the differential equation y'' = y'. **07**

Q.2 **(a)** Using the method of separation of variables solve $u_{xx} = 16\, u_y$. **07**

(b) Find the series solution of the differential equation by Frobenius method **07**

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

OR

(b) (i) Solve $y'' + 4y = 8\cos 2x,\ y(0) = 0,\ y'(0) = 2$ **03**

(ii) Solve $y'' - 4y' - 12y = 7e^{-7x}$ by method of undetermined coefficients. **04**

Q.3 **(a)** Find the Fourier series for the function $f(x) = x^2 + x,\ -\pi \le x \le \pi$. **07**

(b) Find the Fourier series of the function **07**

$$f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$$

OR

Q.3 **(a)** Find the Fourier series with period 3 to represent $f(x) = 2x - x^2$ in the range (0, 3). **07**

(b) Find the half range Fourier cosine series of the function $f(x) = c - x$ in interval (0, c) with period 2c. **07**

Q.4 **(a)** (i) Find the Laplace transform of $e^{-t}(4t^3 + 3\cos 2t + 2e^{-2t})$ **03**

(ii) Prove that **04**

$$L(\sin at) = \frac{a}{s^2 + a^2} \ \text{and}\ L(\cos at) = \frac{s}{s^2 + a^2}$$

$s > 0$, where a is a constant.

(b) Find the Inverse Laplace transform of **07**

(1) $\dfrac{s+3}{(s^2+1)(s^2+9)}$ (2) $\dfrac{2s+3}{s^2-2s+5}$

OR

Q.4 **(a)** (i) Find the Laplace transform of **03**

$$e^{-2t}\int_0^t t\cos t\, dt$$

(ii) Find the Inverse Laplace transform of **04**

$$\frac{1 + e^{-\frac{\pi}{2}s}}{s^2 + 4}$$

(b) Using Laplace transform solve the differential equation $y'' + 6y = 1$, $y(0) = 2$, $y'(0) = 0$ **07**

Q.5 **(a)** (i) Form Partial differential equation by eliminating the arbitrary function from the equation **03**

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$

(ii) Define the following: (1) Beta function (2) Dirac's Delta Function **04**

(b) Express the function as a Fourier Integral **07**

$$f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

OR

Q.5 **(a)** (i) Solve : $p + q = pq$ **03**

(ii) Solve: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. **04**

(b) Solve the following: **07**

(i) $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$

(ii) $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$

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